4 Hydropower, tidal power, and wave power

List of Topics

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Introduction

In this chapter we investigate three different forms of power generation that exploit the abundance of water on Earth: hydropower; tidal power; and wave power. Hydropower taps into the natural cycle of

Solar heat → sea water evaporation → rainfall → rivers → sea.

Hydropower is an established technology that accounts for about 20% of global electricity production, making it by far the largest source of renewable energy. The energy of the water is either in the form of potential energy (reservoirs) or kinetic energy (e.g. rivers). In both cases electricity is generated by passing the water through large water turbines.

Tidal power is a special form of hydropower that exploits the bulk motion of the tides. Tidal barrage systems trap sea water in a large basin and the water is drained through low-head water turbines. In recent years, rotors have been developed that can extract the kinetic energy of underwater currents.

Wave power is a huge resource that is largely untapped. The need for wave power devices to be able to withstand violent sea conditions has been a major problem in the development of wave power technology. The energy in a surface wave is proportional to the square of the amplitude and typical ocean waves transport about 30–70 kW of power per metre width of wave-front. Large amplitude waves generated by tropical storms can travel vast distances across oceans with little attenuation before reaching distant coastlines. Most of the best sites are on the western coastlines of continents between the 40° and 60° latitudes, above and below the equator.

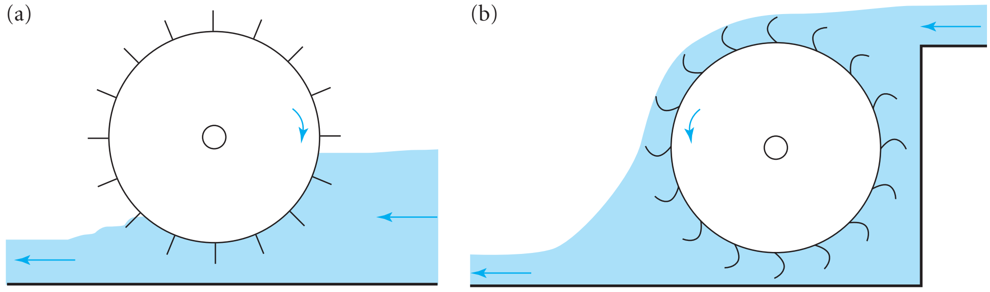

Fig. 4.1 (a) Undershot and (b) overshot waterwheels.

4.1 Hydropower

The power of water was exploited in the ancient world for irrigation, grinding corn, metal forging, and mining. Waterwheels were common in Western Europe by the end of the first millennium; over 5000 waterwheels were recorded in the *Domesday book* of 1086 shortly after the Norman conquest of England. The early waterwheels were of the undershot design (Fig. 4.1(a)) and very inefficient. The development of overshot waterwheels (Fig. 4.1(b)), and improvements in the shape of the blades to capture more of the incident kinetic energy of the stream, led to higher efficiencies.

A breakthrough occurred in 1832 with the invention of the Fourneyron turbine, a fully submerged vertical axis device that achieved efficiencies of over 80%. Fourneyron's novel idea was to employ **fixed guide vanes** that directed water outwards through the gaps between **moving runner blades** as shown in Fig. 4.2. Many designs of water turbines incorporating fixed guide vanes and runners have been developed since. Modern water turbines are typically over 90% efficient.

The main economic advantages of hydropower are low operating costs, minimal impact on the atmosphere, quick response to sudden changes in electricity demand, and long plant life—typically 40 years or more before major refurbishment. However, the capital cost of construction of dams is high and the payback period is very long. There are also serious social

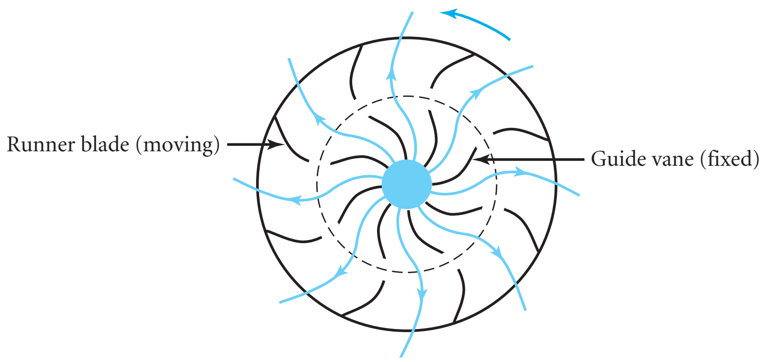

Fig. 4.2 Fourneyron water turbine.

Table 4.1 Installed hydropower

Country		Hydroelectric capacity in 2005 (GW)
USA		80
Canada		67
China		65
Brazil		58
Norway		28
Japan		27
World Total		700
Largest sites for hydropower		
Country	**Site**	**Hydroelectric capacity (GW)**
China	Three Gorges*	18.2
Brazil/Paraguay	Itaipu	12.6
Venezuela	Guri	10.3
USA	Grand Coulee	6.9
Russia	Sayano–Shushenk	6.4
Russia	Krasnoyarsk	6

* Completion due in 2009.

and environmental issues to be considered when deciding about a new hydroelectric scheme, including the displacement of population, sedimentation, changes in water quality, impact on fish, and flooding.

Mountainous countries like Norway and Iceland are virtually self-sufficient in hydropower but, in countries where the resource is less abundant, hydropower is mainly used to satisfy peak-load demand. The hydroelectric capacity by country and the largest sites are shown in Table 4.1.

4.2 Power output from a dam

Consider a turbine situated at a vertical distance h (called the **head**) below the surface of the water in a reservoir (Fig. 4.3). The power output P is the product of the efficiency η, the potential energy per unit volume $\rho g h$, and the volume of water flowing per second Q, i.e.

$$P = \eta \rho g h Q. \tag{4.1}$$

Note that the power output depends on the product hQ. Thus a high dam with a large h and a small Q can have the same power output as a run-of-river installation with a small h

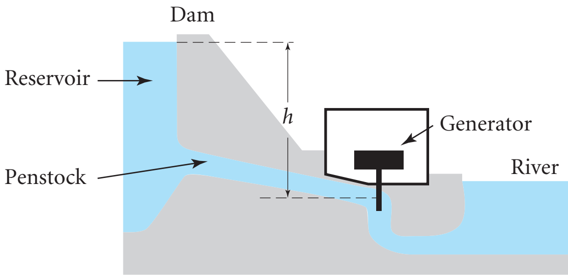

Fig. 4.3 Hydroelectric plant.

and large Q. The choice of which design of water turbine is suitable for a particular location depends on the relative magnitudes of h and Q (see Section 4.7).

EXAMPLE 4.1

Estimate the power output of a dam with a head of 50 m and volume flow rate of 20 m^3s^{-1}. (Assume $\eta = 1, \rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-2}.)

From eqn (4.1) we have $P = \eta\rho g h Q \approx 1 \times 10^3 \times 10 \times 50 \times 20 \approx 10$ MW.

4.3 Measurement of volume flow rate using a weir

For power extraction from a stream it is important to be able to measure the volume flow rate of water. One particular method diverts the stream through a straight-sided channel containing an artificial barrier called a **weir** (Fig. 4.4). The presence of the weir forces the level of the fluid upstream of the weir to rise. The volume flow rate per unit width is related to the height of the undisturbed level of water $y_{\min}$ above the top of the weir by the formula (see Derivation 4.1)

$$Q = g^{1/2}\left(\tfrac{2}{3}y_{\min}\right)^{3/2}. \qquad (4.2)$$

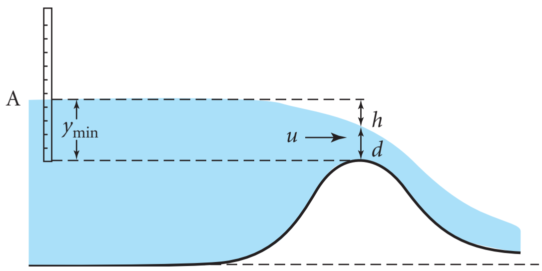

Fig. 4.4 Flow over broad-crested weir.

Derivation 4.1 Flow over a broad-crested weir

Consider a point A on the surface of the water upstream of the weir where the level is roughly horizontal (i.e. $h = 0$ in Fig. 4.4) and the velocity u_A. Towards the weir, the level drops and the speed increases. For a broad-crested weir we can ignore the vertical component of velocity and express the volume rate of water per unit width in the vicinity of the crest in the form

$$Q \approx ud \tag{4.3}$$

where d is the depth of the water near the crest. Using Bernoulli's equation (3.2), noting that the pressure on the surface is constant (atmospheric pressure), we have $\frac{1}{2}u^2 - gh \approx \frac{1}{2}u_A^2$. Hence, if the depth of the water upstream of the weir is much greater than the minimum depth over the crest of the weir, then $u_A^2 \ll u^2$ and $u \approx (2gh)^{1/2}$. Substituting for u in eqn (4.3) we obtain

$$d \approx \frac{Q}{(2gh)^{1/2}}.$$

The vertical distance from the undisturbed level to the top of the weir is $y = d + h$. Substituting for d we have

$$y = \frac{Q}{(2gh)^{1/2}} + h. \tag{4.4}$$

The first term on the right-hand side of (4.4) decreases with h but the second term increases with h. y is a minimum when $dy/dh = 0$, i.e. $-Q/(8gh^3)^{1/2} + 1 = 0$, or

$$h = \left(\frac{Q^2}{8g}\right)^{1/3}. \tag{4.5}$$

Finally, substituting for h from eqn (4.5) in eqn (4.4), yields $y_{\min} = \frac{3}{2}(Q^2/g)^{1/3}$, so that

$$Q = g^{1/2}\left(\tfrac{2}{3}y_{\min}\right)^{3/2},$$

which is known as the Francis formula.

4.4 Water turbines

When water flows through a waterwheel the water between the blades is almost stationary. Hence the force exerted on a blade is essentially due to the difference in pressure across the blade. In a **water turbine**, however, the water is fast moving and the turbine extracts kinetic energy from the water. There are two basic designs of water turbines: impulse turbines and reaction turbines. In an **impulse turbine**, the blades are fixed to a rotating wheel and each blade rotates in air, apart from when the blade is in line with a high speed jet of water. In a **reaction turbine**, however, the blades are fully immersed in water and the thrust on the moving blades is due to a combination of reaction and impulse forces.

An impulse turbine called a **Pelton wheel** is shown in Fig. 4.5. In this example there are two symmetrical jets, and each jet imparts an impulse to the blade equal to the rate of change of

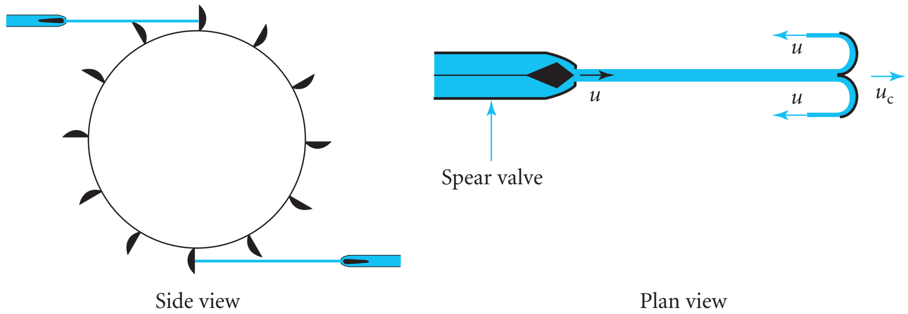

Fig. 4.5 Impulse turbine (Pelton wheel).

momentum of the jet. The speed of the jet is controlled by varying the area of the nozzle using a spear valve. Thomas Pelton went to seek his fortune in the Californian Gold Rush during the nineteenth century. By the time he arrived on the scene the easy pickings had already been taken and the remaining gold had to be extracted from rocks that needed to be crushed. Impulse turbines were being used to drive the mills to grind the rocks into small lumps. Pelton observed the motion of the turbine blades and deduced that not all the momentum of the jets was being utilized. He realized that some momentum was being lost because the water splashed in all directions on striking the blades. He redesigned the cups so that the direction of the splash was opposite to that of the incident jet. This produced a marked improvement in efficiency and Pelton thereby made his fortune.

To calculate the maximum power output from a Pelton wheel, we consider a jet moving with velocity u and the cup moving with velocity u_c. Relative to the cup, the velocity of the incident jet is $(u - u_c)$ and the velocity of the reflected jet is $-(u - u_c)$. Hence the total change in the velocity of the jet is $-2(u - u_c)$. The mass of water striking the cup per second is ρQ, so the force on the cup is given by

$$F = 2\rho Q(u - u_c). \tag{4.6}$$

The power output P of the turbine is the rate at which the force F does work on the cup in the direction of motion of the cup, i.e.

$$P = Fu_c = 2\rho Q(u - u_c)u_c \tag{4.7}$$

To derive the maximum power output we put $dP/du_c = 0$, yielding $u_c = \frac{1}{2}u$.

Substituting in eqn (4.7) then yields the maximum power as

$$P_{max} = \tfrac{1}{2}\rho Q u^2. \tag{4.8}$$

Thus the maximum power output is equal to the kinetic energy incident per second.

As in the Fourneyron turbine (Section 4.1), modern reaction turbines use fixed guide vanes to direct water into the channels between the blades of a runner mounted on a rotating wheel (see Fig. 4.6). However, the direction of radial flow is inward. (In the Fourneyron turbine the outward flow caused problems when the flow rate was either increased or decreased.)

The most common designs of reaction turbines are the Francis turbine and the Kaplan turbine. In a Francis turbine the runner is a spiral annulus, whereas in the Kaplan turbine it is

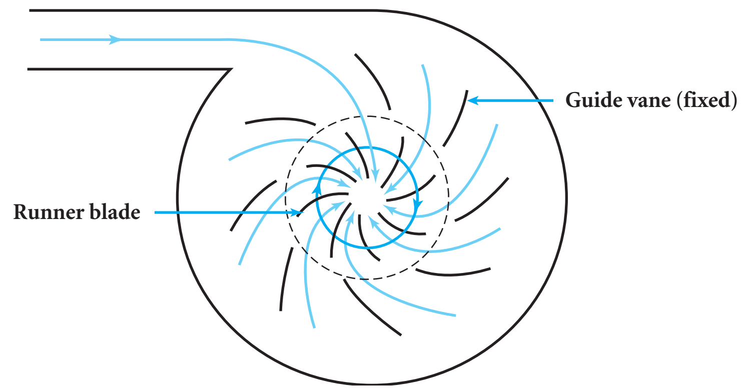

Fig. 4.6 Reaction turbine (plan view).

propeller-shaped. In both designs the kinetic energy of the water leaving the runner is small compared with the incident kinetic energy.

The term 'reaction turbine' is somewhat misleading in that it does not completely describe the nature of the thrust on the runner. The magnitude of the reaction can be quantified by applying Bernoulli's eqn (3.2) to the water entering (subscript 1) and leaving (subscript 2) the runner, i.e.

$$\frac{p_1}{\rho} + \frac{1}{2}q_1^2 = \frac{p_2}{\rho} + \frac{1}{2}q_2^2 + E \tag{4.9}$$

where E is the energy per unit mass of water transferred to the runner. Consider two cases: (a) $q_1 = q_2$, and (b) $p_1 = p_2$. In case (a), eqn (4.9) reduces to

$$E = \frac{p_1 - p_2}{\rho}, \tag{4.10}$$

i.e. the energy transferred arises from the difference in pressure between inlet and outlet. In case (b), E is given by

$$E = \tfrac{1}{2}(q_1^2 - q_2^2) \tag{4.11}$$

i.e. the energy transferred is equal to the difference in the kinetic energy between inlet and outlet. In general we define the **degree of reaction** R as

$$R = \frac{p_1 - p_2}{\rho E} = 1 - \frac{(q_1^2 - q_2^2)}{2E} \tag{4.12}$$

(see Example 4.2).

The velocity diagrams in the laboratory frame of reference for an impulse turbine and a reaction turbine are shown in Fig. 4.7(a) and (b), respectively. The symbols $\boldsymbol{u}$, $\boldsymbol{q}$ and $\boldsymbol{w}$ denote the velocity of the runner blade, the absolute velocity of the fluid, and the velocity of the fluid relative to the blade. Figure 4.7 shows the velocity triangles on the outer radius of the runner

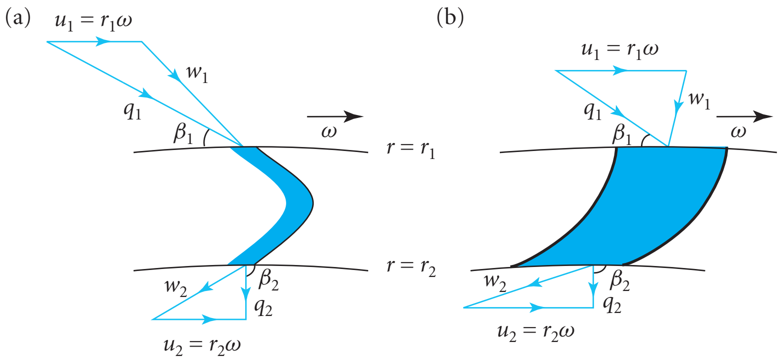

Fig. 4.7 Velocity diagrams for: (a) an impulse turbine; (b) a reaction turbine.

$r = r_1$ and the inner radius $r = r_2$. The runner rotates with angular velocity ω, so that the velocity of the blade is $u_1 = r_1\omega$ on the outer radius and $u_2 = r_2\,\omega$ on the inner radius.

The torque on the runner is

$$T = \rho Q(r_1 q_1 \cos\beta_1 - r_2 u_2 \cos\beta_2).$$

Putting $r_1 = u_1/\omega$ and $r_2 = u_2/\omega$, the work done per second is given by

$$P = T\omega = \rho Q(u_1 q_1 \cos\beta_1 - u_2 q_2 \cos\beta_2).$$

The term in brackets represents the energy per unit mass

$$E = u_1 q_1 \cos\beta_1 - u_2 q_2 \cos\beta_2. \tag{4.13}$$

Equating the incident power due to the head of water h from eqn (4.1) to the power output of the turbine, given by Euler's turbine eqn (3.32), we have

$$\eta\rho g h Q = \rho Q(u_1 q_1 \cos\beta_1 - u_2 q_2 \cos\beta_2).$$

The term $\rho Q u_2 q_2 \cos\beta_2$ represents the rate at which kinetic energy is removed by the water leaving the runner.

We define the hydraulic efficiency as

$$\eta = \frac{u_1 q_1 \cos\beta_1 - u_2 q_2 \cos\beta_2}{gh}. \tag{4.14}$$

The maximum efficiency is achieved when the fluid leaves the runner at right angles to the direction of motion of the blades, i.e. when $\beta_2 = \frac{\pi}{2}$ so that $\cos\beta_2 = 0$. Equation (4.14) then reduces to

$$\eta_{\max} = \frac{u_1 q_1 \cos\beta_1}{gh}. \tag{4.15}$$

EXAMPLE 4.2

Consider a particular reaction turbine in which the areas of the entrance to the stator (the stationary part of the turbine), the entrance to the runner, and the exit to the runner are all equal. Water enters the stator radially with velocity $q_0 = 2\ \mathrm{m\ s^{-1}}$ and leaves the stationary vanes of the stator at an angle $\beta_1 = 10^\circ$ with an absolute velocity $q_1 = 10\ \mathrm{m\ s^{-1}}$. The velocity of the runner at the entry radius $r = r_1$ is u_1 in the tangential direction, and is such that the velocity of the water w_1 relative to the runner is in the radial direction. On leaving the runner, the total velocity is q_2 in the radial direction. Given that the head is $h = 11$ m, calculate the degree of reaction and the hydraulic efficiency.

The volume flow rate is q_0A_0 into the stator, w_1A_1 into the runner, and q_2A_2 out of the runner. Since $A_0 = A_1 = A_2$ it follows by mass conservation that $q_0 = w_1 = q_2$. The energy transfer per unit mass E is given by eqn (4.13). Since the total velocity q_2 leaving the runner is in the radial direction, we have $\beta_2 = \pi/2$. Putting $q_1 \cos\beta_1 = u_1$ then $E = u_1^2$. Also, the square of the total velocity is $q_1^2 = u_1^2 + w_1^2 = u_1^2 + q_2^2$, since w_1 and q_2 are equal and radial. Hence the degree of reaction is $R = 1 - \frac{(q_1^2 - q_2^2)}{2E} = 1 - \frac{u_1^2}{2u_1^2} = \frac{1}{2}$. The hydraulic efficiency is $\eta = u_1 q_1 \cos\beta_1/(gh) \approx 0.90$.

4.4.1 Choice of water turbine

The choice of water turbine depends on the site conditions, notably on the head of water h and the water flow rate Q. Figure 4.8 indicates which turbine is most suitable for any particular combination of head and flow rate. Impulse turbines are suited for large h and a low Q, e.g. fast moving mountain streams. Kaplan turbines are suited for low h and large Q (e.g. run-of-river sites) and Francis turbines are usually preferred for large Q and large h, e.g. dams. A useful parameter for choosing the most suitable turbine is the **shape** (or **type**) **number** S, described in Derivation 4.2.

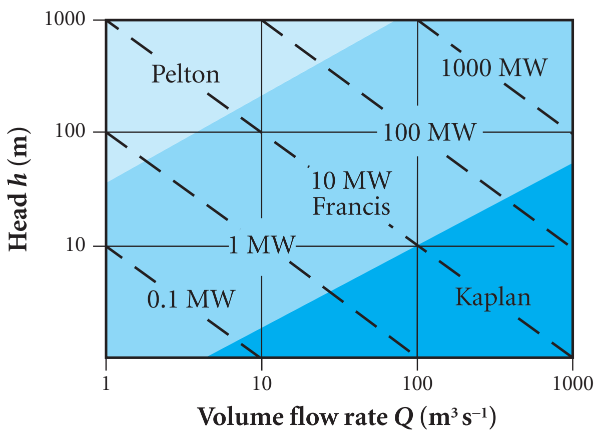

Fig. 4.8 Choice of turbine in terms of head h and volume flow rate Q.

Derivation 4.2 Shape or type number

Dimensional analysis is a useful means for choosing the appropriate type of turbine for a particular combination of h and Q. The power output P from a turbine depends on the head h, the angular velocity ω and diameter D of the turbine, and the density of water ρ. Various dimensionless parameters can be formed from these physical quantities, the power coefficient $K_\mathrm{P} = P/(\omega^3 D^5 \rho)$ and the head coefficient $K_\mathrm{h} = gh/\omega^2 D^2$ being particularly useful ones. When a turbine of a particular design is operating at its maximum efficiency, K_P and K_h will have particular values which can be used to predict the power and head in terms of the diameter D and the angular velocity ω. We can eliminate the dependence on D (which determines the size of the turbine) by forming the dimensionless ratio

$$S = \frac{K_\mathrm{P}^{1/2}}{K_\mathrm{h}^{5/4}} = \frac{\omega P^{1/2}}{\rho^{1/2}(gh)^{5/4}} \tag{4.16}$$

called the shape or type number. Substituting $P = \eta \rho g h Q$ from eqn (4.1) and assuming $\eta = 1$, eqn (4.16) becomes

$$S = \frac{\omega Q^{1/2}}{(gh)^{3/4}}. \tag{4.17}$$

Putting $Q = v_\mathrm{w} A$, $gh = \frac{1}{2} v_\mathrm{w}^2$ (where $A = \pi r^2$ is the inlet area and v_w is the speed of the water) and $\omega = 2\pi f = 2\pi \left(\frac{v_\mathrm{b}}{2\pi R}\right) = \frac{v_\mathrm{b}}{R}$, where v_b is the speed of the blade tip of radius R, we have

$$S = \frac{\frac{v_\mathrm{b}}{R}(v_\mathrm{w}\pi r^2)^{1/2}}{(\frac{1}{2}v_\mathrm{w}^2)^{3/4}} = 2^{3/2}\pi^{1/2}\frac{r}{R}\frac{v_\mathrm{b}}{v_\mathrm{w}} \approx 5\frac{r}{R}\frac{v_\mathrm{b}}{v_\mathrm{w}}. \tag{4.18}$$

For a Pelton turbine $r/R \sim 0.1$, $v_\mathrm{b}/v_\mathrm{w} \approx 0.5$, and $S \sim 0.25$; for a Kaplan turbine $r \sim R$ and $v_\mathrm{b} \sim v_\mathrm{w}$, so $S \sim 5$, while for a Francis turbine $S \sim 1$.

4.5 Impact, economics, and prospects of hydropower

Hydropower sites tend to have a large impact on the local population. Over 1.1 million people were displaced by the Three Gorges dam in China and it has been estimated that 30–60 million people worldwide have had to be relocated due to hydropower. Proposed hydropower plants often provoke controversy and in some countries public opposition to hydropower has stopped all construction except on small-scale projects. Also, dams sometimes collapse for various reasons, e.g. overspilling of water, inadequate spillways, foundation defects, settlement, slope instability, cracks, erosion, and freak waves from landslides in steep-sided valleys around the reservoir. As with nuclear plants, the risk of major accidents is small but the consequences can be catastrophic. Given the long lifetime of dams, even a typical failure rate as low as one per 6000 dam years means that any given dam has a probability of about 1% that it will collapse at sometime in its life. In order to reduce the environmental impact and the consequences of dam failure, the question arises as to whether it is better to build a small number of large reservoirs or a large number of small ones. Though small reservoirs tend to be more acceptable to the public than large ones, they need a much larger total reservoir area than a single large reservoir providing the same volume of stored water.

An argument in favour of hydropower is that it does not produce greenhouse gases or acid rain gases. However, water quality may be affected both upstream and downstream of a dam due to increases in the concentrations of dissolved gases and heavy metals. These effects can be mitigated by inducing mixing at different levels and oxygenating the water by auto-venting turbines. The installation of a hydropower installation can also have a major impact on fish due to changes in the habitat, water temperature, flow regime, and the loss of marine life around the turbines.

The capital cost of construction of hydropower plants is typically much larger than that for fossil fuel plants. Another cost arises at the end of the effective life of a dam, when it needs to be decommissioned. The issue as to who should pay for the cost involved in decommissioning is similar to that for nuclear plants: the plant owners, the electricity consumers, or the general public? On the positive side, production costs for hydropower are low because the resource (rainfall) is free. Also, operation and maintenance costs are minimal and lifetimes are long: typically 40–100 years. The efficiency of a hydroelectric plant tends to decrease with age due to the build-up of sedimentation trapped in the reservoir. This can be a life-limiting factor because the cost of flushing and dredging is usually prohibitive.

The economic case for any hydropower scheme depends critically on how future costs are discounted (see Chapter 11). Discounting reduces the benefit of long-term income, disadvantaging hydropower compared with quick payback schemes such as CCGT generation (see Chapter 2). Hydropower schemes therefore tend to be funded by governmental bodies seeking to improve the long-term economic infrastructure of a region rather than by private capital.

Despite the strong upward trend in global energy demand, the prospects for hydropower are patchy. In the developed world the competitive power market has tilted the balance away from capitally intensive projects towards plants with rapid payback of capital. As long as relatively cheap fossil fuels are available, the growth of hydropower is likely to be limited to parts of the world where water is abundant and labour costs for construction are low. However, it is a source of carbon-free energy and its importance would be enhanced by restrictions on carbon emissions aimed at tackling global warming.

4.6 Tides

There are two high tides and two low tides around the Earth at any instant. One high tide is on the longitude closest to the Moon and the other on the longitude furthest from the Moon. The low tides are on the longitudes at 90° to the longitudes where the high tides are situated. On any given longitude the interval between high tides is approximately 12 hours 25 minutes (see Exercise 4.8). The difference in height between a high tide and a low tide is called the **tidal range**. The mid-ocean tidal range is typically about 0.5–1.0 metres but is somewhat larger on the continental shelves. In the restricted passages between islands and straits the tidal range can be significantly enhanced, up to as much as 12 m in the Bristol Channel (UK) and 13 m in the Bay of Fundy (Nova Scotia). Tidal power has the advantage

Table 4.2 Tidal potential of some large tidal range sites

Country	Site	Mean tidal range (m)	Basin area (km^2)	Capacity (GW)
Argentina	Golfo Nuevo	3.7	2376	6.6
Canada	Cobequid	12.4	240	5.3
India	Gulf of Khambat	7.0	1970	7.0
Russia	Mezen	6.7	2640	15.0
Russia	Penzhinsk	11.4	20530	87.4
UK	Severn	7	520	8.6

over other forms of alternative energy of being predictable. For conventional tidal power generation it is necessary to construct huge tidal basins in order to produce useful amounts of electricity. However, in recent years, an alternative technology for exploiting strong tidal currents has been under development using underwater rotors, analogous to wind turbines. Table 4.2 shows the potential of some large tidal range sites in various locations around the world.

4.6.1 Physical cause of tides

The main cause of tides is the effect of the Moon. The effect of the Sun is about half that of the Moon but increases or decreases the size of the lunar tide according to the positions of the Sun and the Moon relative to the Earth. The daily rotation of the Earth about its own axis only affects the location of the high tides. In the following explanation we ignore the effect of the Sun (see Exercise 4.9).

For simplicity we assume that the Earth is covered by water. Consider a unit mass of water situated at some point P as shown in Fig. 4.9. The gravitational potential due to the Moon is given by

$$\phi = -\frac{Gm}{s}$$

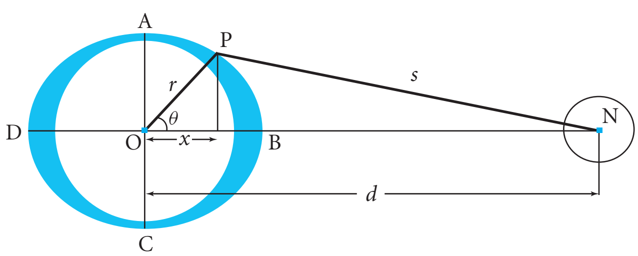

Fig. 4.9 Tidal effects due to the Moon (not to scale).

where G is the gravitational constant, m is the mass of the Moon, and s is the distance from P to the centre of the Moon. For $d \gg r$ we can expand $1/s$ as follows:

$$\frac{1}{s} = \frac{1}{[d^2 + r^2 - 2dr\cos\theta]^{\frac{1}{2}}} = \frac{1}{d}\left[1 + \left(-\frac{2r}{d}\cos\theta + \frac{r^2}{d^2}\right)\right]^{-\frac{1}{2}}$$

$$= \frac{1}{d}\left[1 + \frac{r}{d}\cos\theta + \frac{r^2}{d^2}\left(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\right) + \cdots\right].$$

The first term in the expansion does not yield a force and can be ignored. The second term corresponds to a constant force, Gm/d^2, directed towards N, which acts on the Earth as a whole and is balanced by the centrifugal force due to the rotation of the Earth–Moon system. The third term describes the variation of the Moon's potential around the Earth. The surface profile of the water is an equipotential surface due to the combined effects of the Moon and the Earth. The potential of unit mass of water due to the Earth's gravitation is gh, where h is the height of the water above its equilibrium level and $g = GM/r^2$ is the acceleration due to gravity at the Earth's surface, where M is the mass of the Earth. Hence, the height of the tide $h(\theta)$ is given by

$$gh(\theta) - \frac{Gmr^2}{d^3}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) = 0$$

or

$$h(\theta) = h_{\max}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) \tag{4.19}$$

where

$$h_{\max} = \frac{mr^4}{Md^3} \tag{4.20}$$

is the maximum height of the tide, which occurs at points B and D ($\theta = 0$ and $\theta = \pi$). Putting $m/M = 0.0123$, $d = 384\,400$ km, and $r = 6378$ km we obtain $h_{\max} \approx 0.36$ m, which is roughly in line with the observed mean tidal height.

4.6.2 Tidal waves

There are two tidal bulges around the Earth at any instant. A formula for the speed of a tidal wave in a sea of uniform depth h_0 is obtained from shallow water theory (see Derivation 4.3) as

$$c = \sqrt{gh_0}. \tag{4.21}$$

The tidal bulges cannot keep up with the rotation of the Earth (see Exercise 4.12), so the tides lag behind the position of the Moon, the amount dependent on latitude. The presence of continents and bays significantly disturbs the tides and can enhance their range (see Section 4.9).

Derivation 4.3 Shallow water theory

We consider a wave such that the wavelength λ is much greater than the mean depth of the sea h_0. We also assume that the amplitude of the wave is small compared with the depth, in which case the vertical acceleration is small compared with the acceleration due to gravity, g. Hence the pressure below the surface is roughly hydrostatic and given by

$$p = p_0 + \rho g(h - y) \tag{4.22}$$

where p_0 is atmospheric pressure and $y = h(x, t)$ is the wave profile on the free surface (Fig. 4.10). Differentiating eqn (4.22) with respect to x we have $\partial p/\partial x = -\rho g \partial h/\partial x$. Neglecting second-order terms, the equation of motion in the x-direction is of the form

$$\partial u/\partial t = -g\partial h/\partial x. \tag{4.23}$$

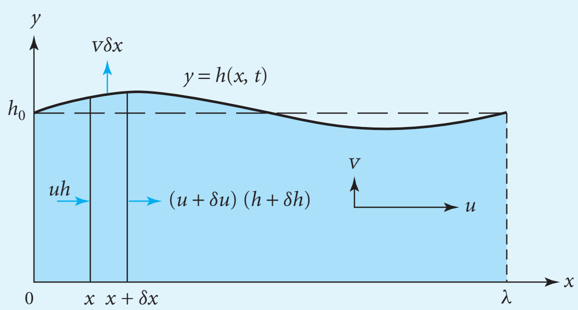

Fig. 4.10 Shallow water wave.

Since $h(x, t)$ is independent of y it follows from (4.23) that u is also independent of y. This allows us to derive an equation of mass conservation in terms of u and h. Consider a slice of fluid between the planes x and x+δx. The volume flowing per second is uh across x and $(u + \delta u)(h + \delta h)$ across x+δx. By mass conservation, the difference in the volume flowing per second from x to x+δx is equal to the volume displaced per second $v\delta x$ in the vertical direction. Hence

$$uh = (u + \delta u)(h + \delta h) + v\delta x.$$

Putting $v = \partial h/\partial t$, $\delta u \approx (\partial u/\partial x)\delta x$, $\delta h \approx (\partial h/\partial x)\delta x$, and noting that $h\partial u/\partial x \gg u\partial h/\partial x$ and $h \approx h_0$, yields the mass continuity equation as

$$-\frac{\partial u}{\partial x} = \frac{1}{h_0}\frac{\partial h}{\partial t}. \tag{4.24}$$

Eliminating u between eqns (4.23) and (4.24) we obtain the wave equation

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 h}{\partial t^2} \tag{4.25}$$

for the height profile $h(x, t)$ of the wave, where $c = \sqrt{gh_0}$ is the wave speed.

4.7 Tidal power

The earliest exploitation of tidal power was in tidal mills, created by building a barrage across the mouth of a river estuary. Sea water was trapped in a tidal basin on the rising tide and released at low tide through a waterwheel, providing power to turn a stone mill to grind corn. Tidal barrages for electricity generation use large low-head turbines and can operate for a greater fraction of the day. An important issue is whether it is better to use conventional turbines that are efficient but operate only when the water is flowing in one particular direction or less efficient turbines that can operate in both directions (i.e. for the incoming and the outgoing tides).

The first large-scale tidal power plant in the world was built in 1966 at La Rance in France. It generates 240 MW using 24 low-head Kaplan turbines. A number of small tidal power plants have also been built more recently in order to gain operational experience and to investigate the long-term ecological and environmental effects of particular locations. Various proposals during the last century to build a large-scale tidal barrage scheme for the River Severn in the UK have been turned down due to the large cost of construction, public opposition and the availability of cheaper alternatives.

4.8 Power from a tidal barrage

A rough estimate of the average power output from a tidal barrage can be obtained from a simple energy balance model by considering the average change of potential energy during the draining process. Consider a tidal basin of area A as shown in Fig. 4.11. The total mass of water in the tidal basin above the low water level is $m = \rho A h$, where h is the tidal range. The height of the centre of gravity is $\frac{1}{2}h$, so the work done in raising the water is $mg(\frac{1}{2}h) = \frac{1}{2}\rho g A h^2$. Hence the average power output (see Example 4.3) is

$$P_{\text{ave}} = \frac{\rho g A h^2}{2T} \tag{4.26}$$

where T is the time interval between tides, i.e. the tidal period. In practice, the power varies with time according to the difference in water levels across the barrage and the volume of water

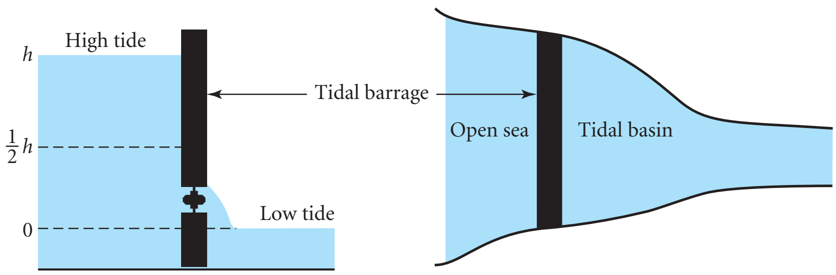

Fig. 4.11 Tidal barrage.

allowed to flow through the turbines. Also, the operating company would seek to optimize revenue by generating electricity during periods of peak-load demand when electricity prices are highest.

EXAMPLE 4.3

Estimate the average power output of a tidal basin with a tidal range of 7 m and an area of tidal basin of 520 km^2 (i.e. Severn barrage).

Substituting in eqn (4.26), noting that the tidal period is $T \approx 4.5 \times 10^4$ s ($T \approx 12.5$ h), the average power output is

$$P_{\text{ave}} = \frac{\rho g A h^2}{2T} \approx \frac{10^3 \times 10 \times 520 \times 10^6 \times 7^2}{2 \times 4.5 \times 10^4} \approx 2.8\,\text{GW}.$$

4.9 Tidal resonance

The tidal range varies in different oceans of the world due to an effect known as **tidal resonance**. For example, the Atlantic Ocean has a width of about 4000 km and an average depth of about 4000 m, so the speed of a shallow water wave eqn (4.21) is about $c = \sqrt{gh_0} \approx \sqrt{10 \times 4000} \approx 200$ m s^{-1}. The tidal frequency is about 2×10^{-5}s^{-1}, so the wavelength is $\lambda = c/f \approx 200/(2 \times 10^{-5})$ m $\approx 10^4$ km. This is about twice the width of the Atlantic and so resonance occurs; the time taken for the shallow water wave to make the round trip, reflecting off both shores, is about the same as the tidal period, so the amplitude builds up.

The wave amplitude also increases on the continental shelf and can reach about 3 m at the shores. River estuaries can also exhibit large tidal resonance if the length and depth of the estuary are favourable. From eqn (4.21) the time taken for a wave to propagate the length of the channel and back to the inlet is given by $t = 2L/c = 2L/\sqrt{gh_0}$ (see Example 4.4). If this time is equal to half the time between successive tides then the tidal range is doubled (see Derivation 4.4).

Derivation 4.4 Tidal resonance in a uniform channel

For simplicity, consider a uniform channel of length L such that the end at $x = 0$ is open to the sea and the other end of the channel at $x = L$ is a vertical wall. Suppose that the height of the incident tidal wave varies with time as $h_\text{i}(t) = a\cos(\omega t)$. We consider a travelling wave of the form

$$h_\text{i}(x,t) = a\cos(kx - \omega t).$$

From the mass continuity eqn (4.24), we have

$$-\frac{\partial u_\text{i}}{\partial x} = \frac{1}{h_0}\frac{\partial h_\text{i}}{\partial t} = \frac{\omega a}{h_0}\sin(kx - \omega t).$$

Integrating with respect to x yields the velocity in the horizontal direction as

$$u_{\mathrm{i}}(x,t) = \frac{\omega a}{h_0 k}\cos(kx - \omega t).$$

In order to satisfy the boundary condition $u = 0$ at $x = L$ (since there cannot be any flow across the barrier) we superimpose a reflected wave of the form

$$u_{\mathrm{r}}(x,t) = \frac{\omega a}{h_0 k}\cos(kx + \omega t).$$

The total velocity at $x = L$ is given by $u(L,t) = u_{\mathrm{i}}(L,t) + u_{\mathrm{r}}(L,t)$ $= \frac{\omega a}{h_0 k}[\cos(kL - \omega t) + \cos(kL + \omega t)] = \frac{2\,\omega a}{h_0 k}\cos(kL)\cos(\omega t) = 0$. Hence $kL = \frac{\pi}{2}(2n+1)$, and the lowest mode of oscillation ($n = 0$) is given by $kL = \frac{\pi}{2}$. Putting $k = 2\pi/\lambda$ we then obtain the minimum length of the channel as $L = \frac{\lambda}{4}$. The total height of the incident and reflected waves is

$$h(x,t) = h_{\mathrm{i}}(x,t) + h_{\mathrm{r}}(x,t) = a\cos(kx - \omega t) - a\cos(kx + \omega t) = 2a\sin(kx)\sin(\omega t).$$

At the end of the channel, $x = L$, the height is $h(L,t) = 2a\sin(\omega t)$, i.e. double that due to the incident wave. This causes the amplitude to build up with the result that the tidal range can be very large—in the River Severn estuary between England and Wales a range of 10–14 m is observed.

EXAMPLE 4.4

An estuary has an average depth of 20 m. Estimate the length of estuary required for tidal resonance.

Equating the time for a wave to travel the length of the channel and back again to half the tidal period we have $\frac{2L}{\sqrt{10\times 20}} = \frac{1}{2}\times 4.5\times 10^4$, so that $L \approx 160$ km.

4.10 Kinetic energy of tidal currents

In particular locations (e.g. between islands) there may be strong tidal currents that transport large amounts of kinetic energy. In recent years various devices for extracting the kinetic energy have been proposed. These devices are essentially underwater versions of wind turbines and obey the same physical principles as those described in Chapter 5. In the majority of designs the axis of rotation of the turbine is horizontal and the device is mounted on the seabed or suspended from a floating platform. Before installation, the tidal currents for any particular location need to be measured to depths of 20 m or more in order to determine the suitability of the site. The first generation of prototype kinetic energy absorbers have been operated in shallow water (i.e. 20–30 m) using conventional engineering components. Later generations are likely to be larger, more efficient, and use specially designed low-speed electrical generators and hydraulic transmission systems.

4.11 Ecological and environmental impact of tidal barrages

The installation of a tidal barrage has a major impact on both the environment and ecology of the estuary and the surrounding area for the following reasons.

1. The barrage acts as a major blockage to navigation and requires the installation of locks to allow navigation to pass through.
2. Fish are killed in the turbines and impeded from migrating to their spawning areas.
3. The intertidal wet/dry habitat is altered, forcing plant and animal life to adapt or move elsewhere.
4. The tidal regime may be affected downstream of a tidal barrage. For example, it has been claimed that a proposed barrage for the Bay of Fundy in Canada could increase the tidal range by 0.25 m in Boston, 1300 km away.
5. The water quality in the basin is altered since the natural flushing of silt and pollution is impeded, affecting fish and bird life.

On the positive side, there are the benefits arising from carbon-free energy, improved flood protection, new road crossings, marinas, and tourism.

4.12 Economics and prospects for tidal power

Large tidal barrages have the economic disadvantages of large capital cost, long construction times, and intermittent operation. On the other hand, they have long plant lives (over 100 years for the barrage structure and 40 years for the equipment) and low operating costs. An alternative idea is to create a closed basin in the estuary known as a **tidal lagoon**. The wall of a tidal lagoon does not extend across the whole channel so the environmental effects are lessened and the impact on fish and navigation is reduced. Also, by restricting the tidal lagoons to shallow water, the retaining wall can be low and cheap to build. The global tidal resource has been estimated as 3000 GW, but only 3% of this is in areas where the tidal range is enhanced and hence suitable for power generation. So far, large barrage schemes have not been pursued.

The economics of small tidal current devices (kinetic energy absorbers) has the attraction that they can be installed on a piecemeal basis, thereby reducing the initial capital outlay. They also have a more predictable output than wind turbines and there is no visual impact. The danger to fish is minimal because the blades rotate fairly slowly (typically about 20 revolutions per minute). The potential for tidal stream generation around the UK has been estimated as possibly 10 GW, a contribution of about a quarter to the UK electricity demand.

The long-term economic potential and environmental impact of such devices will become clearer after trials on various designs, notably in the UK, Canada, Japan, Russia, Australia, and China. The engineering challenge is to design reliable and durable equipment capable of operating for many years in a harsh marine environment with low operational and maintenance costs.

4.13 Wave energy

The waves on the surface of the sea are caused mainly by the effects of wind. The streamlines of air are closer together over a crest and the air moves faster. It follows from Bernoulli's theorem (3.2) that the air pressure is reduced, so the amplitude increases and waves are generated. As a wave crest collapses the neighbouring elements of fluid are displaced and forced to rise above the equilibrium level (Fig. 4.12).

The motion of the fluid beneath the surface decays exponentially with depth. About 80% of the energy in a surface wave is contained within a quarter of a wavelength below the surface. Thus, for a typical ocean wavelength of 100 m, this layer is about 25 m deep. We now derive an expression for the speed of a surface wave using intuitive physical reasoning. The water particles follow circular trajectories, as shown in Fig. 4.12 (See Exercise 4.16).

Consider a surface wave on deep water and choose a frame of reference that moves at the wave velocity, c, so that the wave profile remains unchanged with time. Noting that the pressure on the free surface is constant (i.e. atmospheric pressure), Bernoulli's eqn (3.2) yields

$$u_c^2 - u_t^2 - 2gh = 0 \tag{4.27}$$

where u_c is the velocity of a particle at a wave crest, u_t is the velocity of a particle at a wave trough, and h is the difference in height between a crest and a trough. If r is the radius of a circular orbit and τ is the wave period then we can put

$$u_c = \frac{2\pi r}{\tau} - c, \quad u_t = -\frac{2\pi r}{\tau} - c, \quad h = 2r. \tag{4.28}$$

Substituting for u_c, u_t and h from eqn (4.28) in eqn (4.27), and putting $\lambda = c\tau$, we obtain the wave speed as

$$c = \sqrt{g\lambda/(2\pi)}. \tag{4.29}$$

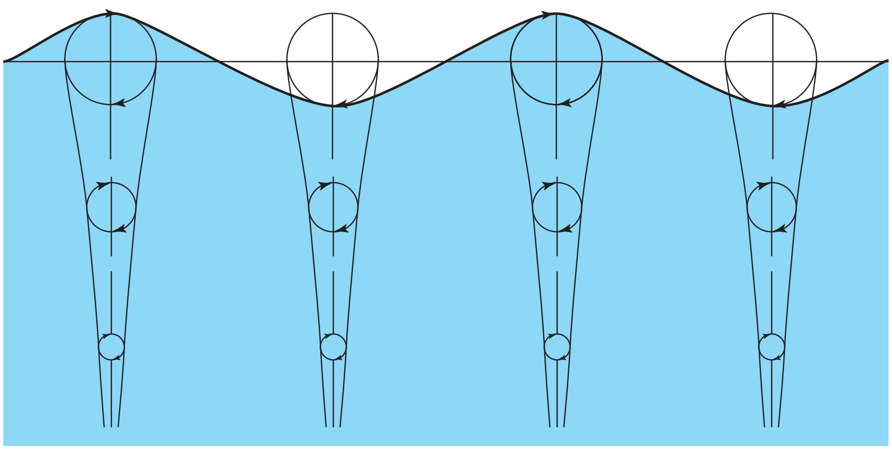

Fig. 4.12 Surface wave on deep water.

It follows from eqn (4.29) that the wave speed increases with wavelength, so that surface waves are **dispersive**. In practice the wave profile on the surface of the sea is a superposition of waves of various amplitudes, speeds, and wavelengths moving in different directions. The net displacement of the surface is therefore more irregular than that of a simple sine wave. Hence, in order for a wave power device to be an efficient absorber of wave energy in real sea conditions, it needs to be able to respond to random fluctuations in the wave profile.

The total energy E of a surface wave per unit width of wave-front per unit length in the direction of motion is given by

$$E = \tfrac{1}{2}\rho g a^2 \tag{4.30}$$

(see Derivation 4.5). The dependence of wave energy on the square of the amplitude has mixed benefits. Doubling the wave amplitude produces a fourfold increase in wave energy. However, too much wave energy poses a threat to wave power devices and measures need to be taken to ensure they are protected in severe sea conditions.

The power P per unit width in a surface wave is the product of E and the group velocity c_g, given by

$$c_g = \tfrac{1}{2}\sqrt{g\lambda/(2\pi)} \tag{4.31}$$

(see Exercise 4.11). Hence the incident power per unit width of wave-front (Example 4.5) is

$$P = \tfrac{1}{4}\rho g a^2\sqrt{g\lambda/(2\pi)}. \tag{4.32}$$

In mid-ocean conditions the typical power per metre width of wave-front is 30–70 kW m^{-1}.

EXAMPLE 4.5

Estimate the power per unit width of wave-front for a wave amplitude $a = 1$ m and wavelength of 100 m.

From eqn (4.32), the power per unit width of wave-front is

$$P = \tfrac{1}{4}\rho g a^2\sqrt{\tfrac{g\lambda}{2\pi}} \approx \tfrac{1}{4} \times 10^3 \times 10 \times 1^2 \times \sqrt{\tfrac{10\times 10^2}{2\times 3.14}} \approx 32\ \text{kW m}^{-1}.$$

Derivation 4.5 Energy in a surface wave

Consider unit width of a wave with a surface profile of the form

$$z = a\sin\left(\frac{2\pi x}{\lambda}\right)$$

as shown in Fig. 4.13. (The time-dependence is irrelevant for this derivation.) The gain in potential energy of an elemental mass $\delta m = \rho g\ \delta x\ \delta z$ of fluid in moving from $-z$ to $+z$ is $\delta V = \delta m g(2z) = 2\rho g z\ \delta x\ \delta z$. Hence the total potential energy of the elevated section of fluid is

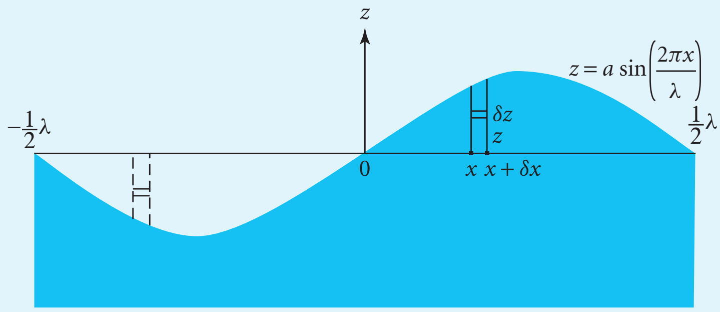

Fig. 4.13 Energy of surface wave.

$$V = 2\rho g\int_{x=0}^{x=\frac{1}{2}\lambda}\int_{z=0}^{z=a\sin(2\pi x/\lambda)} z\,\mathrm{d}z\,\mathrm{d}x = \rho g a^2\int_{x=0}^{x=\frac{1}{2}\lambda}\sin^2(2\pi x/\lambda)\mathrm{d}x = \tfrac{1}{4}\rho g a^2\lambda.$$

Assuming equipartition of energy, the average kinetic energy is equal to the average potential energy, so that the total energy over a whole wavelength is $E = \frac{1}{2}\rho g a^2\lambda$, or

$$E = \tfrac{1}{2}\rho g a^2$$

per unit length in the x-direction and per unit width of wavefront.

4.14 Wave power devices

Though the first patent for a wave power device was filed as early as 1799, wave power was effectively a dormant technology until the early 1970s, when the world economy was hit by a series of large increases in oil prices. Wave power was identified as one of a number of sources of alternative energy that could potentially reduce dependency on oil. It received financial support to assess its technical potential and commercial feasibility, resulting in hundreds of inventions for wave power devices, but most of these were dismissed as either impractical or uneconomic. The main concerns were whether wave power devices could survive storms and their capital cost. During the 1980s, publicly funded research for wave power virtually disappeared as global energy markets became more competitive. However, in the late 1990s interest in wave power technology was revived due to increasing evidence of global climate change and the volatility of oil and gas prices. A second generation of wave power devices emerged, which were better designed and had greater commercial potential.

In general, the key issues affecting wave power devices are:

- survivability in violent storms;
- vulnerability of moving parts to sea water;
- capital cost of construction;
- operational costs of maintenance and repair;
- cost of connection to the electricity grid.

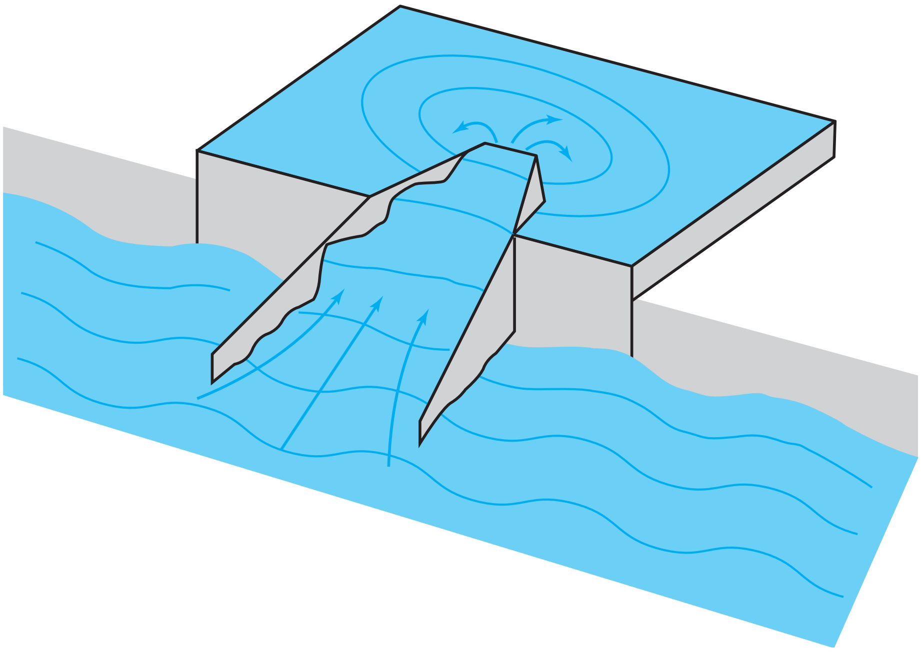

Fig. 4.14 TAPCHAN.

We now describe various types of wave power device and examine how they operate and how they address the above challenges.

4.14.1 Spill-over devices

TAPCHAN (TAPered CHANnel) is a Norwegian system in which sea waves are focused in a tapered channel on the shoreline. Tapering increases the amplitude of the waves as they propagate through the channel. The water is forced to rise up a ramp and spill over a wall into a reservoir about 3–5 m above sea level (Fig. 4.14). The potential energy of the water trapped in the reservoir is then extracted by draining the water back to the sea through a low-head Kaplan turbine. Besides the turbine, there are no moving parts and there is easy access for repairs and connections to the electricity grid. Unfortunately, shore-based TAPCHAN schemes have a relatively low power output and are only suitable for sites where there is a deep water shoreline and a low tidal range of less than about a metre. To overcome these limitations, a floating offshore version of TAPCHAN called Wave Dragon is under development, with an inlet span of around 200 m, to generate about 4 MW.

4.14.2 Oscillating water columns

The oscillating water column (OWC) uses an air turbine housed in a duct well above the water surface (Fig. 4.15). The base of the device is open to the sea, so that incident waves force the water inside the column to oscillate in the vertical direction. As a result the air above the

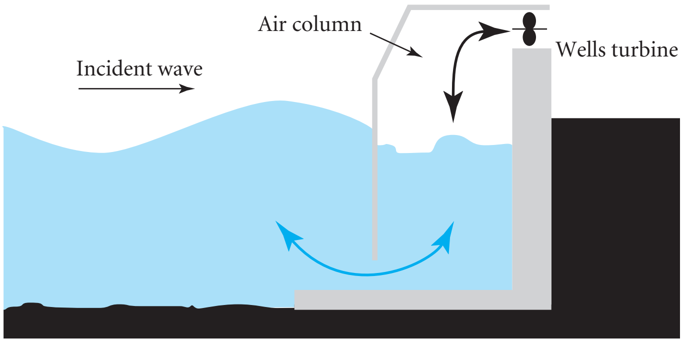

Fig. 4.15 Oscillating water column (OWC).

surface of the water in the column moves in phase with the free surface of the water inside the column and drives the air turbine. The speed of air in the duct is enhanced by making the cross-sectional area of the duct much less than that of the column.

A key feature of the OWC is the design of the air turbine, known as the **Wells turbine**. It has the remarkable property of spinning in the same direction irrespective of the direction of air flow in the column! Unlike conventional turbine blades, the blades in a Wells turbine are symmetrical about the direction of motion (Fig. 4.16). Relative to a blade, the direction of air flow is at a non-zero angle of attack α. The net force acting on the blade in the direction of motion is then given by

$$F = \mathcal{L}\sin\alpha - \mathcal{D}\cos\alpha \tag{4.33}$$

where $\mathcal{L}$ and $\mathcal{D}$ are the lift and drag forces acting on the blade. It is clear from the force diagram in Fig. 4.16(b) that the direction of the net force is the same, irrespective of whether the air is flowing upwards or downwards inside the air column.

The shape of the blade is designed such as to maximize the net force on the blade and the operational efficiency of a Wells turbine is around 80%. At low air velocities the turbine absorbs power from the generator in order to maintain a steady speed of rotation, whilst for

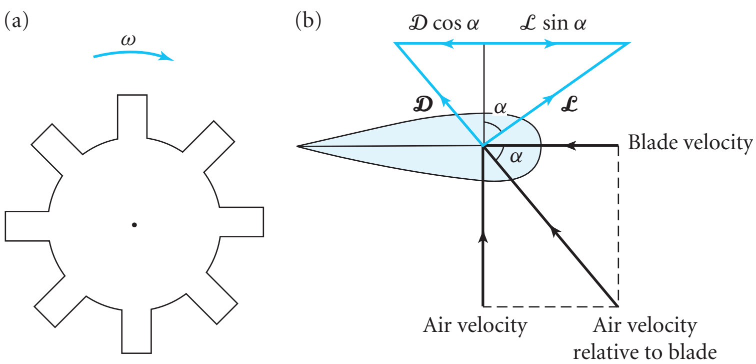

Fig. 4.16 Wells turbine. (a) Plan view of blades; (b) velocity and force triangles in frame of reference of a blade.

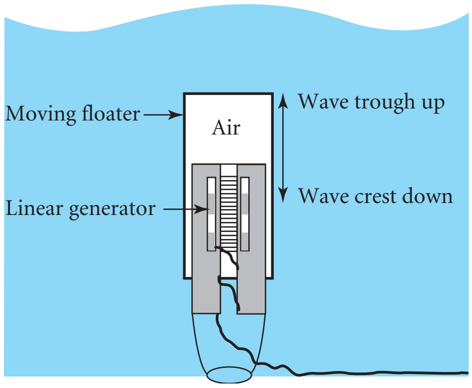

Fig. 4.17 Archimedes Wave Swing.

large air velocities the air flow around the blades is so turbulent that the net force in the direction of motion of the blade becomes erratic and the efficiency is reduced.

Two designs of shore-based OWCs are the **Limpet** (UK) and the **Osprey** (UK); generating about 0.5 and 1.5 MW, respectively. A prototype 0.5 MW Australian OWC scheme is also being developed, which uses a 40 m wide parabolic wave reflector to focus waves onto a 10 m wide shoreline OWC; the capital cost is 30% higher but the output is increased by 300%. A large floating OWC known as the **Mighty Whale** has been developed in Japan. It generates 110 kW but its primary role is as a wave breaker to produce calm water for fisheries and other marine activities.

4.14.3 Submerged devices

Submerged devices have the advantage of being able to survive despite rough sea conditions on the surface. They exploit the change in pressure below the surface when waves pass overhead: the pressure is increased for a wave crest but is decreased in the case of a wave trough. An example of this type of device is the **Archimedes Wave Swing** (AWS, Fig. 4.17). The AWS is a submerged air-filled chamber (the 'floater'), 9.5 m in diameter and 33 m in length, which oscillates in the vertical direction due to the action of the waves. The motion of the floater energizes a linear generator tethered to the sea bed. The AWS has the advantage of being a 'point' absorber, i.e. it absorbs power from waves travelling in all directions, and extracts about 50% of the incident wave power. Also, being submerged at least 6 m below the surface it can avoid damage in violent sea conditions on the surface. The device has the advantages of simplicity, no visual impact, quick replacement and cost effectiveness in terms of the power generated per kg of steel. A pre-commercial pilot project off the coast of Portugal has three AWS devices and produces 8 MW. A fully commercial AWS system could involve up to six devices per kilometre and it is estimated that the global potential for AWS is around 300 GW.

4.14.4 Floating devices

In the early 1970s public interest in wave power was stimulated by a novel device known as the **Salter duck** (Fig. 4.18). The device floated on water and rocked back and forth with

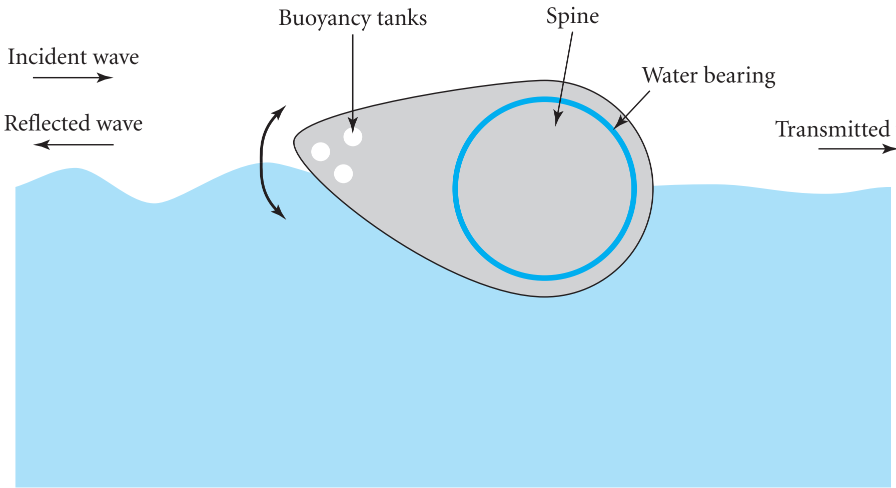

Fig. 4.18 Salter duck.

the incident waves. The shape was carefully chosen such that the surface profile followed the circular trajectories of water particles, so that most of the incident wave energy was absorbed with only minimal reflection and transmission. Efficiencies of around 90% were achieved in ideal conditions. The complete system envisaged a string of Salter ducks of several kilometres in total length parallel to a wave-front. A spinal column, of 14 m diameter, used the relative motion between each duck and the spine to provide the motive force to generate power. The device was designed to be to be used in the Atlantic Ocean for wavelengths of the order of 100 m but never got beyond small-scale trials due to lack of funding in the 1980s when governmental support for wave power in the UK was dropped in favour of wind power. Nonetheless, the Salter duck provides a useful benchmark for comparing the efficiencies of all wave power devices.

A much more recent type of floating device is the **Pelamis** (Fig. 4.19). It is a semi-submerged serpentine construction consisting of series of cylindrical hinged segments that are pointed towards the incident waves. As waves move along the device, the segments rock back and forth and the relative motion between adjacent segments activates hydraulic rams that pump high pressure oil through hydraulic motors and drive electrical generators. A three-segment version of Pelamis is 130 m long and 3.5 m in diameter and generates 750 kW. The combination of great length and small cross-section to the incident waves provides good protection to large amplitude waves. Three Pelamis devices are due to be installed 5 km off the coast of Portugal and it is estimated that about 30 machines per square kilometre would generate about 30 MW. In order to prevent unwanted interference effects, the devices are spaced apart by about 60–90 m.

Fig. 4.19 Pelamis.

4.15 Environmental impact, economics, and prospects of wave power

As with most forms of alternative energy, wave power does not generate harmful greenhouse gases. Opposition to shore-based sites could be an issue in areas of scenic beauty, on account of the visual impact (including the connections to the electricity transmission grid) and the noise generated by air turbines in the case of oscillating water columns. The visual impact is much less significant for offshore devices but providing cables for electricity transmission to the shore is an added cost.

The global potential of wave power is very large, with estimates of 1–10 TW. Around the UK the Department of Trade and Industry (DTI) estimated (2001) a potential of about 6 GW from wave power. The main challenges for the implementation of wave power are to reduce the capital costs of construction, to generate electricity at competitive prices, and to be able to withstand extreme conditions at sea. Wave power is generally regarded as a high risk technology. Moving to shore-based and near-shore devices reduces the vulnerability to storms but the power available is less than that further out at sea. Even the largest floating devices are vulnerable in freak storms: every 50 years in the Atlantic Ocean there is a wave with an amplitude about ten times the height of the average wave, so any device must be able to withstand a factor of a hundred times the wave energy. Measures to combat such conditions such as submerging the devices can provide an effective means of defence but add to the cost of the system. Another factor to consider is that the frequency of incident sea waves is only about 0.2 Hz, much lower than the frequency of 50–60 Hz for electricity transmission. Though this not a difficult electrical engineering problem, the challenge is to find cost-effective solutions.

Wave power is beginning to look competitive in certain niche markets, especially in remote locations where electricity supplies are expensive. However, it is likely to take one or two decades to gather sufficient operational experience for wave power to compete with other alternative energy technologies. In the long term as fossil fuel reserves become scarce, and concerns over global warming increase, forecasts of an eventual global potential for wave power to provide about 15% of total electricity production do not seem unreasonable, as part of a diverse mix of alternative energy sources.

SUMMARY

- The power output P from a dam is $P = \eta \rho g h Q$.
- The dimensionless shape number $S = \omega Q^{1/2}/(gh)^{3/4} \approx 2^{3/2}\pi^{1/2}(r/R)(v_b/v_w)$ is a useful parameter in choosing the most suitable type of turbine for a particular combination of head h and volume flow rate Q.
- The volume flow rate per unit width over a weir is given by $Q = g^{1/2}(\frac{2}{3}y_{\min})^{3/2}$, where $y_{\min}$ is the height of the undisturbed level of water above the top of the weir.
- The Fourneyron water turbine employed fixed guide vanes to direct water radially outwards into the gaps between moving runner blades, and was over 80% efficient.

- In an impulse turbine the thrust arises from the momentum imparted by high speed water jets striking the cups. In a Pelton wheel the cups are shaped so that the jets splash in the opposite direction to the incident jet, in order to maximize the transfer of momentum.
- In a reaction turbine the blades are fully immersed in water. Fixed guide vanes direct the water into the gaps between the blades of a runner. The thrust is due to a combination of reaction and impulse forces.
- Combining the formula for the power output of a dam and Euler's turbine equation yields the hydraulic efficiency of a turbine as $\eta = (u_1 q_1 \cos\beta_1 - u_2 q_2 \cos\beta_2)/(gh)$.
- Hydroelectric installations have a high capital cost but low operational costs. There are environmental as well as significant social, safety, and economic issues but the electricity is almost carbon-free.
- The main cause of tides is the variation of the gravitational attraction of the Moon around the surface of the Earth. There are two tidal bulges, one facing the Moon and the other diametrically opposite.
- The speed of a tidal wave in a sea of uniform depth h_0 is given by $c = \sqrt{gh_0}$.
- The average power output of a tidal barrage is $P_{\text{ave}} = \rho g A h^2/(2T)$.
- The height of the tides can be increased by tidal resonance, due to the superposition of incident and reflected waves.
- Tidal power from barrages has limited potential, mainly due to the lack of suitable locations, the high capital cost, and the environmental impact.
- Tidal stream projects that use underwater rotors to absorb the kinetic energy of water currents are an alternative means of exploiting tidal power and have considerable potential.
- The total energy E of a surface wave per unit width of wave-front per unit length in the direction of motion is given by $E = \frac{1}{2}\rho g a^2$. About 80% of the energy is contained within a quarter of a wavelength from the surface.
- The power per unit width of wave-front is $P = \frac{1}{4}\rho g a^2\sqrt{\frac{g\lambda}{2\pi}}$. In mid-ocean conditions the typical power per metre width of wave-front is 30–70 kW m^{-1}.
- Wave power is a vast natural resource but serious issues need to be resolved, especially survivability in storms and capital cost.
- Some shore-based wave power schemes (such as TAPCHAN and the oscillating water column) have been shown to be feasible for small-scale operation.
- Large-scale submerged and floating devices (e.g. the Archimedes Wave Swing and Pelamis) can generate much more power and are undergoing sea trials prior to commercial development.

FURTHER READING

Acheson, D. J. (1990). *Elementary fluid dynamics*. Clarendon Press, Oxford. Good account of shallow water and deep water waves.

Boyle, G. (ed.) (2004). *Renewable energy*, 2nd edn. Oxford University Press, Oxford. Good qualitative description and case studies.

Douglas, J.F., Gasiorek, J.M., and Swaffield, J.A. (2001). *Fluid mechanics*. Prentice-Hall Englewood Cliffs, NJ. Textbook on fluid mechanics—good discussion of dimensional analysis and of turbines.

Kuznetsov, N., Moz'ya, V., and Vainberg, B. (2002). *Linear water waves: a mathematical approach*. Cambridge University Press, Cambridge. Advanced textbook, including modelling of submerged devices.

WEB LINKS

www.worldenergy.org Useful data and overview of current developments.

LIST OF MAIN SYMBOLS

a	wave amplitude	R	degree of reaction
c	wave speed	S	shape factor
E	energy	t	time
g	acceleration due to gravity	u, v	velocity components
G	gravitational constant	V	potential energy
h	head	x, y, z	coordinates
k	wave number	β	angle
P	power	η	efficiency
p	pressure	λ	wavelength
q	total velocity	ρ	density
Q	volume flow rate	ω	angular velocity

EXERCISES

4.1 Check the units to verify the expression $P = \eta\rho g h Q$ for the power output from a dam.

4.2 Estimate the power output of a dam with a head $h = 100$ m and volume flow rate $Q = 10\ \mathrm{m^3\ s^{-1}}$. (Assume efficiency is unity, $\rho = 10^3\ \mathrm{kg\ m^{-3}}$, $g = 9.81\ \mathrm{m\ s^{-2}}$.)

4.3 Assuming the volume flow rate per unit width over a weir is of the form $Q = g^a y_{\mathrm{min}}^b$, use dimensional analysis to determine the numerical values of a and b.

4.4 Draw a sketch of an impulse turbine consisting of four jets.

4.5 Verify that the power output of an impulse turbine is a maximum when $u_c = \frac{1}{2}u$, and that the maximum power delivered to the cup is given by $P_{\mathrm{max}} = \frac{1}{2}\rho Q u^2$.

4.6 Explain how a rotary lawn sprinkler works.

4.7 Discuss who should pay for the cost involved in decommissioning dams when they reach the end of their life.

4.8 A turbine is required to rotate at 6 r.p.m. with a volume flow rate of 5 $m^3\ s^{-1}$ and a head of 30 m. What type of turbine would you choose?

4.9 If there are two high tides around the Earth at any instant, explain why the interval between successive high tides is 12 hours 25 min rather than 12 hours.

4.10* Compare the magnitude of the effect of Sun on the tides: (a) when the Sun and Moon are both on the same side of the Earth; (b) when the Sun and the Moon are on opposite sides of the Earth. ($m_{\mathrm{Sun}} = 2 \times 10^{30}$ kg, $m_{\mathrm{Moon}} = 7.4 \times 10^{22}$ kg, $d_{\mathrm{Sun}} = 1.5 \times 10^{11}$m, $d_{\mathrm{Moon}} = 3.8 \times 10^{8}$m.)

4.11* (a) Show by substitution that the profile $h = a\cos(kx - \omega t) + b\cos(kx + \omega t)$ satisfies the tidal wave equation $\partial^2 h/\partial x^2 = (1/c^2)\partial^2 h/\partial t^2$. (b) A uniform channel of length L is bounded at both ends by a vertical wall. Derive the height and velocity profiles of shallow water waves in the channel.

4.12 Show that the speed of a tidal bulge on the equator in the Atlantic Ocean (depth ~4000 m) is less than the speed, due to the Earth's rotation, of the seabed.

4.13 Assuming the speed c of surface waves on deep water depends only on the acceleration due to gravity g and the wavelength λ, use dimensional analysis to derive an algebraic expression of the form $c = kg^a\lambda^b$, where k is a dimensionless constant.

4.14 Calculate the speed of a surface wave on deep water of wavelength $\lambda =$ 100 m.

4.15 Given that the phase velocity and group velocity of a surface wave are $c = \sqrt{g\lambda}$ and $c_g = \mathrm{d}\omega/\mathrm{d}k$, respectively, where ω is the angular velocity and $k = 2\pi/\lambda$, prove that the group velocity is given by $c_g = \frac{1}{2}\sqrt{g\lambda/(2\pi)}$.

4.16* Consider a two-dimensional surface wave of amplitude a and wavelength λ such that $a \ll \lambda$ on a sea of depth $d \gg \lambda$. Assume the velocity can be expressed in the form $\mathbf{u} = \nabla\phi$, where ϕ is the velocity (called the velocity potential) satisfying Laplace's equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0.$$

(a) Show that travelling wave solutions exist of the form

$$\phi = Ae^{-\frac{2\pi y}{\lambda}}\sin\frac{2\pi}{\lambda}(x - ct).$$

(b) Hence show that the velocity components are given by

$$u = \frac{2\pi}{\lambda}Ae^{-\frac{2\pi y}{\lambda}}\cos\frac{2\pi}{\lambda}(x - ct) \qquad v = -\frac{2\pi}{\lambda}Ae^{-\frac{2\pi y}{\lambda}}\sin\frac{2\pi}{\lambda}(x - ct).$$

(c) Prove that particles of fluid rotate in circles of radius $r = ae^{-\frac{2\pi y}{\lambda}}$.

(d) Prove that the kinetic energy per unit width

$$E = \frac{1}{2}\rho\int_0^{\lambda}\int_0^{\infty}(u^2 + v^2)\mathrm{d}y\mathrm{d}x = \tfrac{1}{4}\rho g a^2\lambda.$$

4.17 An oscillating water column has an air duct of cross-sectional area 1 m^2 and a water duct of cross-sectional area 10 m^2. If the average speed of the water is 1 m s^{-1} calculate the average speed of the air.

4.18 Discuss whether it is better to build a large number of small dams or one large dam.